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2. Sea $f(x,y) = (1-x)(1-2x)(1-y)(1-2y)$, obtén:

i) Máximos y mínimos relativos de $f(x,y)$.
ii) Máximos y mínimos absolutos de $f(x,y)$ en $D = [0,1] \times [0,1]$.

i) $\frac{\partial f}{\partial x} = \left[-(1-2x) - 2(1-x)\right](1-y)(1-2y)$

$$\begin{cases} (*) & \frac{\partial f}{\partial x} = (-3+4x)(1-y)(1-2y) = 0 \\ (**) & \frac{\partial f}{\partial y} = (-3+4y)(1-x)(1-2x) = 0 \end{cases}$$

$(*)$ soluciones $\nearrow$ $x = \frac{3}{4}$, $y = 1$, $y = 1/2$

Si $x = 3/4 \Rightarrow$ solución de $(**)$ es $y = 3/4$
$P_1\left(\frac{3}{4}, \frac{3}{4}\right)$

Si $y = 1 \nearrow x = 1, x = 1/2 \Rightarrow P_2(1,1), P_3(1/2, 1)$

Si $y = \frac{1}{2} \to x = 1, x = \frac{1}{2} \Rightarrow P_4\left(1, \frac{1}{2}\right), P_5\left(\frac{1}{2}, \frac{1}{2}\right)$

$H(x,y)$ $\nearrow f_{xx} = 4(1-y)(1-2y)$
$\to f_{yy} = 4(1-x)(1-2x)$
$\searrow f_{xy} = f_{yx} =$

$f_x = (1-y)(1-2y)(-3+4x)$

$f_{xy} = \left[-1\cdot(1-2y) - 2\cdot(1-y)\right]\cdot(-3+4x)$

$f_{xy} = (-3+4y)(-3+4x)$

$$H(x,y) = \begin{pmatrix} 4(1-y)(1-2y) & (-3+4y)(-3+4x) \\ (-3+4y)(-3+4x) & 4(1-x)(1-2y) \end{pmatrix}$$

$P_1(3/4, 3/4)$

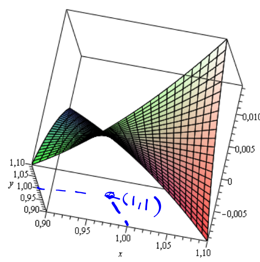
$\hookrightarrow H(3/4, 3/4) = \begin{pmatrix} -1/2 & 0 \\ 0 & -1/2 \end{pmatrix}$

$|H(3/4, 3/4)| = 1/4 > 0$
$f_{xx} < 0$ $\Rightarrow (3/4, 3/4)$ máximo relativo

$P_2(1,1)$

$H(1,1) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$|H(1,1)| < 0 \Rightarrow$ punto de silla

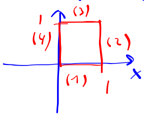


$(1/2, 1) \to$ P.S.
$(1, 1/2) \to$ P.S.
$\left(\frac{1}{2}, \frac{1}{2}\right) \to$ P.S.

ii) $\Omega = \{(x,y) \in \mathbb{R}^2 / [0,1]\times[0,1]\}$

· Relativos $\in \mathring{\Omega} \Rightarrow (3/4, 3/4)$

· $\bar{\Omega}$ (frontera)



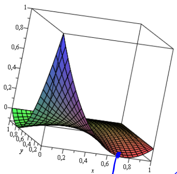
(1) $\begin{cases} y=0 \\ x\in[0,1] \end{cases}$

$\hookrightarrow f(x,0) = (1-x)(1-2x)$

$f'(x) = -(1-2x) - 2(1-x) =$

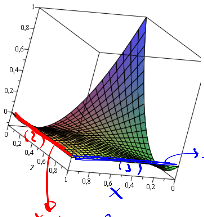
$= -3+4x = 0 \Rightarrow x = 3/4$

$x = 3/4,\ y=0 \rightarrow (3/4, 0)$



(1) $\rightarrow (3/4, 0)$

(3) $\begin{cases} y=1 \\ x\in[0,1] \end{cases} \Rightarrow f(x,1) = 0$



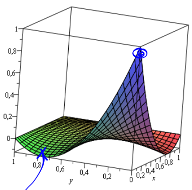
$\rightarrow f = 0$

$x=1 \Rightarrow f=0$ en la zona (2) igual que en (3)

(4) $\begin{cases} x=0 \\ y\in[0,1] \end{cases} \Rightarrow f(0,y) = (1-y)(1-2y)$

$\frac{df}{dy} = -3+4y = 0 \Rightarrow y = 3/4$

$(0, 3/4)$



$(0, 3/4)$

· Otros candidatos son las 4 esquinas (no se detectan con derivadas)

$(0,0), (0,1), (1,0), (1,1)$

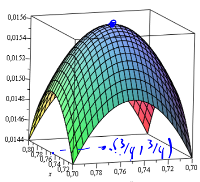
Ahora comparamos la "altura" de todos

· $f(0,0) = 1$

$f(0,1) = f(1,0) = f(1,1) = 0$

$f(0,\frac{3}{4}) = f(\frac{3}{4},0) = -1/8$

$f(3/4, 3/4) = 1/64 \simeq 0{,}0156$



$\rightarrow (3/4, 3/4)$

El (0,0) es el máximo absoluto y $(0,3/4), (3/4,0)$ son los mínimos absolutos